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ELASTICITY OF ROCK IN UNIAXIAL STRAIN

FINAL REPORT

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The pressure calculated in a shock experiment for Westerly granite would be up to 30 percent higher if the appropriate Poisson's ratio for uniaxial strain is used instead of the commonly tabulated value found at high hydrostatic pressure.

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INTRODUCTION

Deformation under the constraint that two principal strains remain zero is called uniaxial strain. Strain is uniaxial in some modes of plane wave and shock wave propagation as well as in the uniform loading of bodies of large lateral extent. Both situations are of some geologic interest; shock loading of rocks is used, for example, in studies of the equation of state. It is often assumed that deformation in upper parts of the earth is uniaxial.

In a recent laboratory study [1] some fifteen different rocks were loaded in uniaxial strain to obtain a better understanding of the role of mineralogy and porosity in their deformation, and to compare any failure with that in conventional triaxial experiments. The rocks fell into two groups, those of low porosity, for which deformation even to very high stress was recoverable, and those of high porosity, which underwent permanent compaction. Behavior of the second group is described in [1, 2]. Here, we explore the extent to which

the recoverable deformation of the first class could have been predicted from independent measurement of elastic properties. Particular attention is given to Poisson's ratio, for this is readily obtainable from the uniaxial strain experiment. We use here the 'tangent' value of Poisson's ratio, determined from differential changes in deformation, rather than the 'secant' value. Thus, Poisson's ratio in uniaxial strain is calculated by means of the relationship found from elastic theory, $\nu = d\sigma_3/(d\sigma_1 + d\sigma_3)$, where the subscripts 1 and 3 refer to the axial and lateral directions, respectively. Similarly, Poisson's ratio in uniaxial compression is $-d\varepsilon_3/d\varepsilon_1$, where $d\varepsilon$ is the differential change in strain.

Review of previous work and many experimental details are given in [1]. The method used in [1] was nearly identical with that of Brown and Swanson [3]. A jacketed cylindrical sample was so loaded by axial stress and confining pressure that the circumferential (and therefore the radial) strain measured by a strain gauge remained zero. A maximum of 10 kb confining pressure could be applied; at this limit the axial stress reached a value of 12 to 30 kb, depending on rock type. The axial stress σ_1, the radial stress σ_3, and the axial strain ε_1 were recorded during the experiment. Permanent strain following a cycle of loading was noted, either from strain gauge readings or, for very large strains, from measurement of the sample dimensions.

We describe here additional experiments on those rocks which showed negligible permanent deformation. These tests were carried out to investigate the extent to which recoverable behavior indicated true elastic deformation. The rocks, total porosity, and modal analysis are listed in Table 1. Volume compressibility and Poisson's ratio were determined as a function of confining pressure, using the same samples and strain gauges as for uniaxial loading. When these data were compared with appropriate parameters from uniaxial loading, Poisson's ratio was found to differ significantly from that of the uniaxial experiment. We suggest below an explanation for this, based on microcracks, following analysis developed in earlier studies [4, 5] of the effect of cracks on elasticity of rock.

2. EXPERIMENTAL OBSERVATIONS

Volumetric compressions to 10 kb pressure, following the procedure outlined in [6], are listed in Table 2. Volumetric compressions have been assumed to be three times the measured axial linear compressions. Where the rock is relatively isotropic this will be close to the actual volumetric compression; where the rock is anisotropic, it will not be, but it is probably the most appropriate quantity to compare with ε_1 from the uniaxial strain experiment.

TABLE 1. ROCKS STUDIED

Rock	Porosity %	Modal analysis
Diabase, II Frederick, Md.	0.1	49 an_{45}, 46 pyr, 3 ox, 2 mica
Gabbro San Marcos, Cal.	0.2	70 an_{42}, 12 mica, 8 pyr, 7 am, 3 ox
Schist Source unknown	0.3	40 qu, 26 mica, 15 or, 7 an_{5}, 7 gar, 5 ox
White marble Source unknown	0.3	99 ca
Granite Barre, Vt.	0.6	26 qu, 25 or, 37 an_{10}, 12 mica
Granite Westerly, R.I.	0.9	27.5 qu, 35.4 mi, 31.4 an_{17}, 4.9 mica

Abbreviations:

qu	quartz	mica	mica, clay
or	orthoclase	gar	garnet
ca	calcite	ox	oxides
pyr	pyroxene	mi	microcline
an	plagioclase		

TABLE 2. VOLUME COMPRESSIONS

Units are 10^{-}

Rock	Pressure, kb								
	0.6	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6
Diabase	8.8	17.3	33.6	49.2	64.6	79.7	94.4	109	124
Gabbro	17.5	41.7	55.0	76.5	97.2	117	137	156	175
Schist	24.5	40.7	68.4	95.1	120	144	168	188	210
Marble	20.1	32.8	51.9	69.5	86.8	104	120	137	153
Barre Granite	26.3	42.8	69.4	93.2	116	137	157	177	198
Westerly Granite	19.9	35.0	62.5	88.0	113	137	161	184	207

Samples of five of the rocks were set up just as for the uniaxial strain experiment [1]. An axial load of several kilobars was applied at a number of different confining pressures starting at about 1 kb. Both axial and circumferential strains were observed, enabling ratio to be determined as a function of pressure to 10 kb. The values, listed in Table 3, have an uncertainty of about 10 percent.

3. ANALYSIS

Before comparing elastic parameters from the uniaxial strain experiment with intrinsic values, we consider what the former ought to be, based on the model used in previous analyses [5], namely a homogeneous elastic matrix containing an isotropic network of cracks. We will consider the role these cracks play in material loaded in such a way that strain is uniaxial. In particular, compressibility and Poisson's ratio will be calculated to see how they depend on crack parameters.

The effective elastic properties of the model under uniaxial strain are found conveniently by applications of Betti's reciprocal theorem. We assume only that the matrix is linearly elastic and that a sufficient number of cracks of small enough size exist that the overall deformation appears to be homogeneous. The body with a single crack or section of a crack acted upon by the axially symmetric stress

TABLE 3. POISSON'S RATIO

Rock	Pressure, kb							
	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6
Diabase	0.29	0.29	0.29	0.30	0.29	0.29	0.31	0.30
Gabbro	0.26	0.26	0.26	0.26	0.27	0.27	0.28	--
Marble	0.20	0.29	0.29	0.30	0.30	0.31	0.29	0.30
Barre Granite	0.27	0.24	0.24	0.25	0.26	0.26	0.24	0.26
Westerly Granite	0.21	0.22	0.22	0.22	0.23	0.23	0.23	0.23

distribution ($d\sigma_1$, $d\sigma_3$, $d\sigma_3$), is shown in Fig. 1a. To find the effective lateral strain, we consider the same body to be loaded on both external and internal surfaces by the stresses (0, $d\sigma_{u3}$, $d\sigma_{u3}$), as shown in the other diagram in Fig. 1b. The reciprocal theorem gives the equality

$$d\sigma_1 d\varepsilon_{u1} + 2d\sigma_3 d\varepsilon_{u3} = 2d\sigma_{u3} d\varepsilon_3 - \int d\tau_{u3} d\delta_f \, dA \tag{1}$$

where $d\tau_{u3}$ is the shear component of $d\sigma_{u3}$ in the plane of the crack, and the integration is over all interior surfaces. We introduce the requirement that $d\varepsilon_3 = 0$ for uniaxial strain and the identities

$$d\tau_{u3} = \sin\beta\cos\beta d\sigma_{u3}$$

$$d\varepsilon_{u3} = (1-\upsilon)\ d\sigma_{u3}/E$$

$$d\varepsilon_{u1} = -2\upsilon\ d\sigma_{u3}/E \tag{2}$$

where υ and E are Poisson's ratio and Young's modulus of the matrix material. Equation 1 can now be written

$$d\sigma_3/d\sigma_1 = \upsilon/(1-\upsilon) + I \tag{3}$$

where $$2I(1-\upsilon) = E \int \sin\beta\cos\beta (d\delta_f/d\sigma_1) dA$$

Note that I is a positive quantity. Brace [1] calculated Poisson's ratio from the slope of plots of σ_3 versus σ_1 by

means of the relation

$$d\sigma_3/d\sigma_1 = \upsilon_{eff}/(1 - \upsilon_{eff}) \tag{4}$$

where υ_{eff} is the effective Poisson's ratio of the porous sample. Combining (3) and (4) and rearranging gives

$$\upsilon_{eff} = \upsilon + (1-\upsilon)I/(2 + I) \tag{5}$$

Thus, υ_{eff} must be greater than the intrinsic value ν because the second term on the right-hand side of (5) is positive.

The stiffness is found in a similar way, except that a uniform axial stress $d\sigma_{u1}$ is applied as in Fig. 1c. One finds, following the steps above, that the effective axial compliance $d\varepsilon_1/d\sigma_1$ is

$$d\varepsilon_1/d\sigma_1 = 1/E - 2\upsilon d\sigma_3/Ed\sigma_1 + I \tag{6}$$

Introducing (3) into (6) gives

$$d\varepsilon_1/d\sigma_1 = (1 + \upsilon)/3K(1 - \upsilon) + I/3K \tag{7}$$

The first term on the right-hand side of equation (7) is the axial compliance of a non-porous body under uniaxial strain; the second term I/3K is greater than zero, so the effective compliance is increased as a result of slip on interior surfaces.

Note that $d\varepsilon_1$ is the change in volumetric strain because the other components of strain are zero here.

An expression for the volumetric strain involving the average pressure can be found by applying the reciprocal theorem to the stress state in Fig. 1a and a uniform pressure dp_u applied to exterior and interior surfaces as in Fig. 1d. The work balance from the reciprocal theorem is

$$d\sigma_1\, d\varepsilon_{u1} + d\sigma_3(d\varepsilon_{u2} + d\varepsilon_{u3}) = dp\ (d\varepsilon_1 + 2d\varepsilon_3) \tag{8}$$

There is no contribution in equation (8) frcm slip on closed cracks because pressure dp_u is normal to this displacement. If cracks are distributed isotropically,

$$d\varepsilon_{u1} = d\varepsilon_{u2} = d\varepsilon_{u3} = dp/3K \tag{9}$$

Introducing the identities

$$dV = d\varepsilon_1 + 2d\varepsilon_3$$

$$dp_{avg} = (d\sigma_1 + 2d\sigma_3)/3$$

we find

$$dV/Vdp_{avg} = 1/K \tag{10}$$

Thus, the compressibility determined in uniaxial strain when

all cracks are closed is equal to the intrinsic value. This result can be verified by means of equations (5) and (7).

4. DISCUSSION

The theory above predicts the behavior of rocks containing an isotropic distribution of cracks under uniaxial strain. Poisson's ratio, when applied compressive stresses are high enough that virtually all cracks are closed, should, based on equation (5), be greater than the intrinsic value ν because of the relative motion between crack faces which may occur when applied stresses are nonhydrostatic. Poisson's ratio at low stress when cracks are open has been shown theoretically to be less than the intrinsic value ν [4]. Thus, one would predict that Poisson's ratio under uniaxial strain for crystalline rock containing cracks should increase from a value less than ν to a value greater than ν as applied stress increases. The magnitude of the increase cannot be evaluated easily; it depends in a complicated way upon the number of cracks and their frictional characteristics, neither of which are now known with certainty from independent measurements.

We test the above prediction for Barre granite, diabase, gabbro, and marble in Figs. 2 and 3. Apparently the values of Poisson's ratio in uniaxial strain (from [1]) do in fact exceed the intrinsic values (from Table 3) for marble, granite, and gabbro at high stress. At low stress the reverse holds for

gabbro, granite, and diabase. The theory cannot be tested against the data for diabase at high stress or marble at low stress because the differences between Poisson's ratio in the two tests is less than the uncertainty in the measurements.

The very high values of Poisson's ratio at high stress for the marble in uniaxial strain suggest that additional factors than motion on cracks may be significant for this material. The most likely is that plastic flow of calcite has occurred; this would probably act in the same sense as motion on cracks and cause Poisson's ratio to increase. Some microscopic evidence presented in [1] suggests that flow of calcite has in fact occurred.

One can carry this comparison one step further for Westerly granite. In a previous analysis [4] for the condition of uniaxial stress, it was also found that Poisson's ratio should exceed the intrinsic value; comparison of that analysis with the present shows that Poisson's ratio under uniaxial strain should be less than the value under uniaxial stress, because slip on cracks is inhibited by the lateral stresses in the former. Thus, Poisson's ratio for the proposed model should have the following characteristics: at low stress, Poisson's ratio is less than the intrinsic value, but equal for all modes of deformation. The value increases as stress increases to a value above the intrinsic value. Poisson's ratio in this range depends upon the stress state, with higher values associated with conditions of less lateral constraint.

Measurements on Westerly granite seem to bear out this prediction. Shown in Fig. 4 are values of Poisson's ratio as a function of axial stress under conditions of uniaxial stress, uniaxial strain, and hydrostatic pressure. At high stress, Poisson's ratio in uniaxial strain is less than the value in uniaxial stress and greater than the value under hydrostatic pressure, as predicted by the theory. The values at zero stress are less than the intrinsic value, as predicted, although the values do not appear to be equal. This discrepancy may be due to differences between individual samples, or to lack of precision in the measurements; both effects are more significant at low stress than at high stress.

Turning next to compressibility, we find that the theory above predicts that volumetric strain per unit average pressure is equal to the intrinsic compressibility for any state of applied stress which is high enough to close all cracks. This equality, which is a property of an ideally elastic material, was unexpected here because the model we analyzed is not truly elastic in this region; stiffness in the axial direction is less than the intrinsic value and Poisson's ratio is greater, for instance.

The model does behave elastically at low stress when all cracks are open, so volumetric strain per unit average pressure equals compressibility in this region, as well. Compressibility is less than the intrinsic value, however, because of the presence of open cracks. At intermediate stresses, where

cracks normal to the maximum compression are closed and those more parallel are open, the rock is elastically anisotropic, and the equality does not hold. Plots of average pressure, σ_1, versus volumetric strain, ε_1, under uniaxial strain and under hydrostatic pressure should therefore have the following characteristics. The curves coincide at low stresses where all cracks are open, and begin to diverge as stress is increased. At high stress the curves are parallel but not necessarily coincident.

In Fig. 5, measurements from [1] are compared with intrinsic volume compressions from Table 2. At high stress, data in uniaxial strain for all rocks except marble lie approximately parallel to the corresponding curve from tests under hydrostatic pressure. Results for marble deviate farthest from predicted behavior, perhaps because of true plastic flow in calcite grains which may have occurred in this experiment. In previous tests, cracks in typical rocks seem to be completely closed by pressures of 2-4 kb, and most of the effects resulting from their closure are small after 1 kb. The early stages of deformation where closing of cracks is important, therefore, cannot be analyzed in plots on the scale of that in Fig. 3. Note, however, that the separation between data from the two tests is not large, suggesting that the effect of anisotropy at low stress upon later deformation is small.

We conclude that behavior of the low porosity rocks was not strictly elastic in the uniaxial strain experiments [1],

even though they did appear to recover in an elastic fashion. Although volumetric compression could have been predicted from independent measurement of elastic properties (Fig. 5) Poisson's ratio could not (Figs. 2, 3, and 4). Presumably other elastic parameters which involve deviatoric stress or strain, such as Young's modulus and possibly compressional wave velocity, would have shown similar disagreement.

Nonuniqueness of Poisson's ratio has important bearing on investigations of the equation of state of rocks because shock data is obtained from experiments which apparently are in effect uniaxial strain tests at high strain rate. Poisson's ratio must be known in order to reduce this data to obtain plots of average pressure versus volume, and generally a value measured in tests under hydrostatic pressure has been used. Values of Poisson's ratio of 0.23 and 0.18, corresponding to measurements at high and low pressure, have been used in reducing data on Westerly granite, to cite only one example. The average pressure is 15% to 30% higher if the value of 0.30 obtained in uniaxial strain by Brace [1] is used.

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FIGURE CAPTIONS

Figure 1 Body containing a closed crack loaded by appropriate states of stress for applying the reciprocal theorem.

(a) The given applied stresses ($d\sigma_1$, $d\sigma_3$, $d\sigma_3$) produce slip of $d\delta_f$ on a closed internal surface. In uniaxial strain, $d\varepsilon_3=0$.

(b) A uniform stress (0, $d\sigma_3$, $d\sigma_3$) is applied to external and internal surfaces to find the effective lateral compliance.

(c) A uniform stress ($d\sigma u_1$,0,0) is applied to external and internal surfaces to find the effective axial compliance.

(d) A uniform pressure dp_u is applied to internal and external surfaces to find the volumetric strain resulting from ($d\sigma_1$, $d\sigma_3$, $d\sigma_3$).

Figure 2 Poisson's ratio for marble and Barre granite versus lateral stress, σ_3, under uniaxial strain (open circles) and hydrostatic compression (closed circles). Error bars show the typical uncertainty in the measurements. From [1].

Figure 3 Poisson's ratio for gabbro and diabase versus lateral stress, σ_3, under uniaxial strain (open circles) and hydrostatic pressure (closed circles). Error bars show the typical uncertainty in the measurements. From [1].

Figure 4 Effective Poisson's ratio $\bar{\upsilon}$ for Westerly granite versus axial stress σ_1 under uniaxial stress [4], uniaxial strain [1], and hydrostatic pressure [1]. The sample used in the uniaxial strain and hydrostatic pressure tests was different from the sample used in the uniaxial stress test. The error bars show the typical uncertainty in the measurements.

Figure 5 Volumetric compression compared for hydrostatic pressure and uniaxial strain. Curves show pressure, P, versus volumetric strain, θ, from Table 2. Circles give mean stress, $\bar{\sigma}$, versus volumetric strain, ε_1, from tests under uniaxial strain [1]. Probable error in both sets of measurements was less than the diameter of the circles.

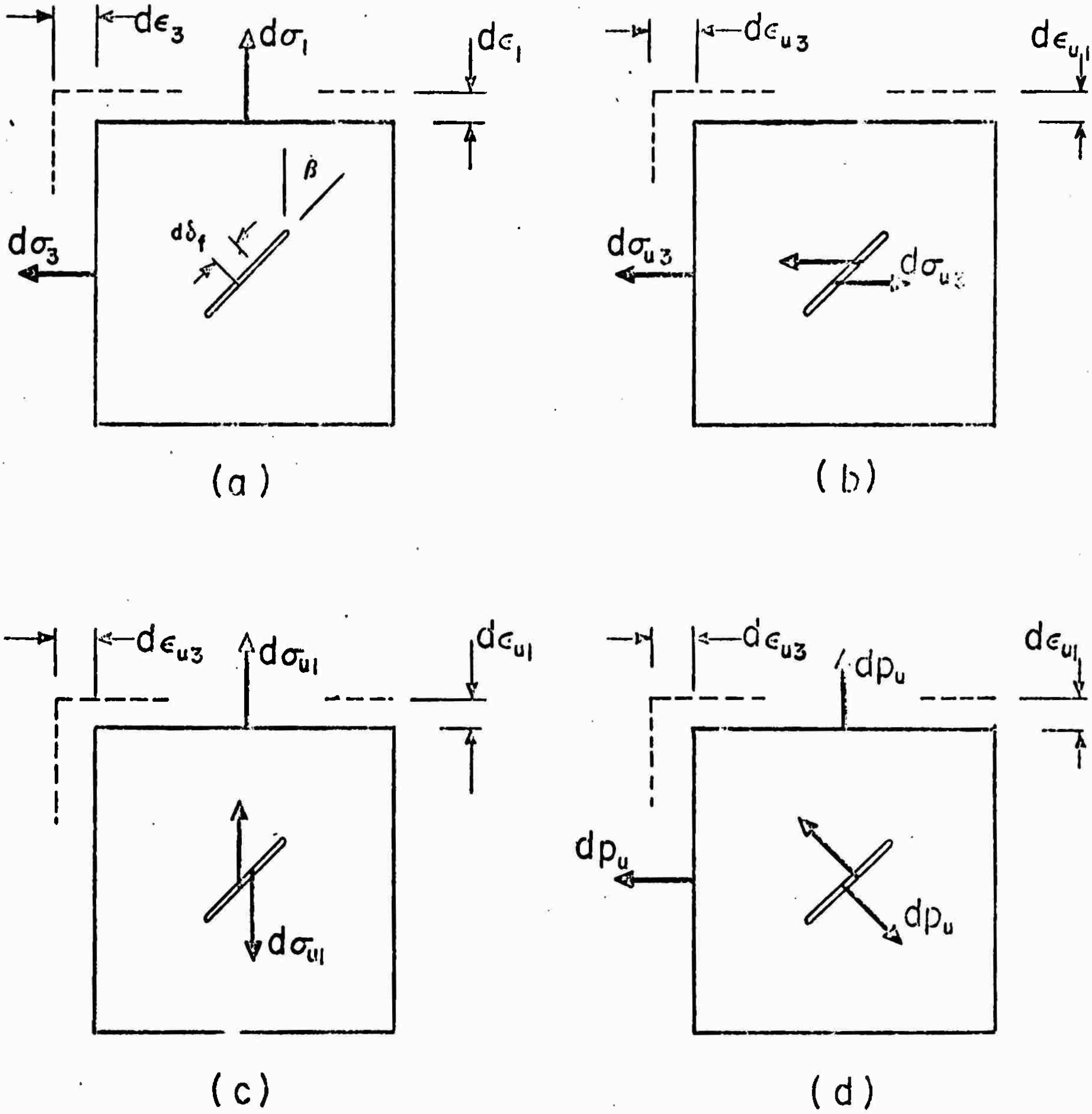

Fig. 1 Body containing a closed crack loaded by appropriate states of stress for applying the reciprocal theorem

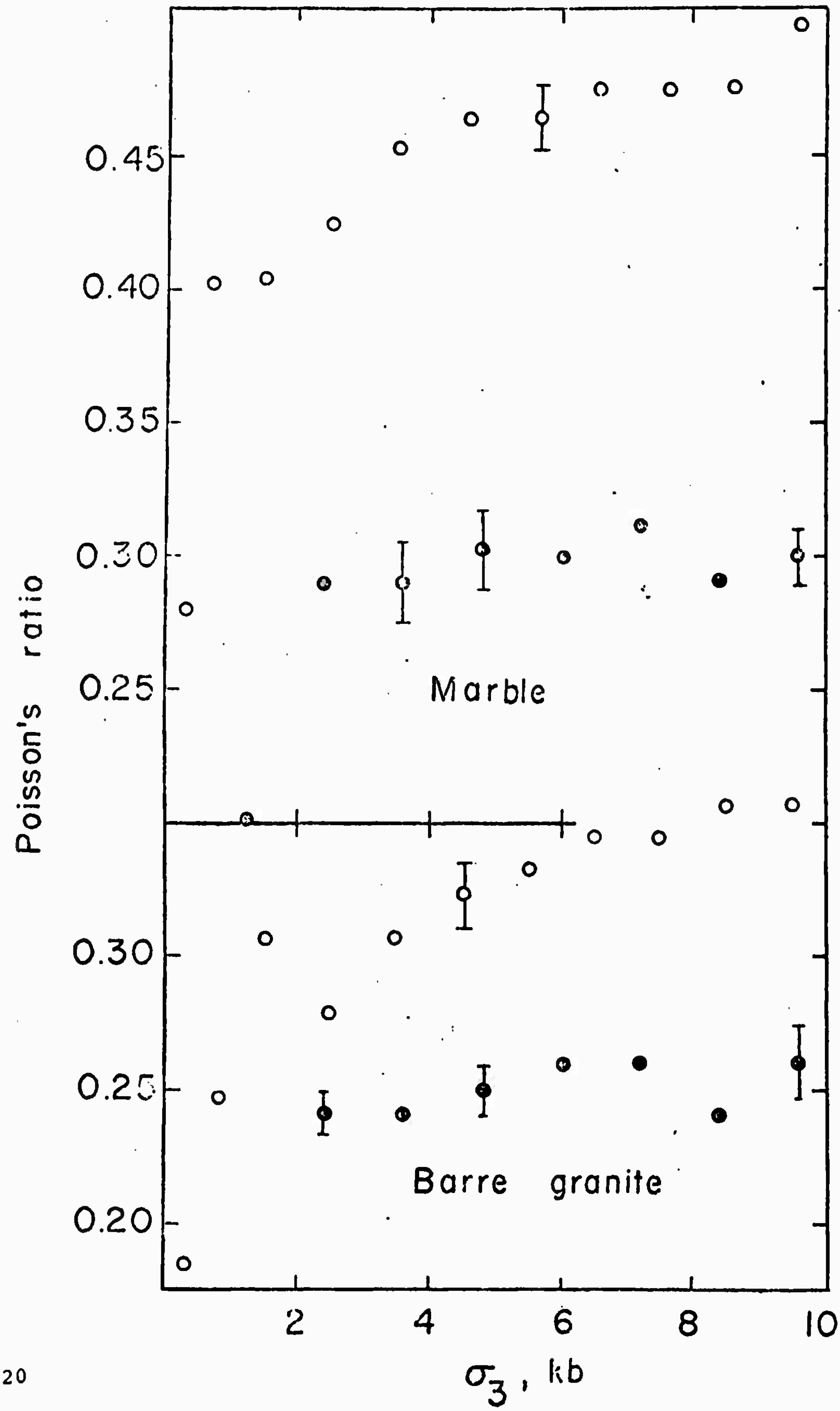

Fig. 2 Poisson's ratio for marble and Barre granite *versus* lateral stress, σ_3, under uniaxial strain (open circles) and *hydrostatic* compression (closed circles)

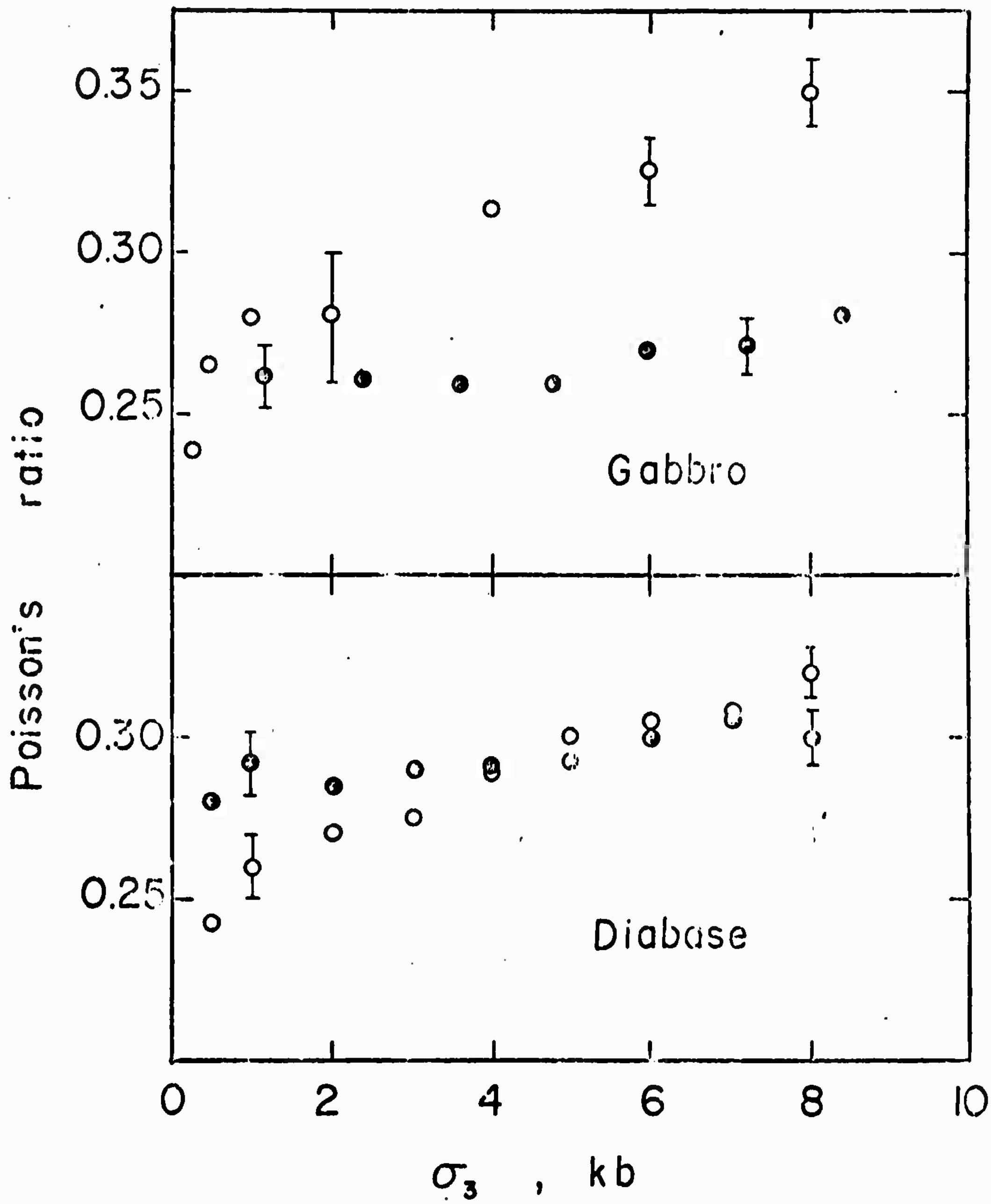

Fig 3. Poisson's ratio for gabbro and diabase <u>versus</u> lateral stress, σ_3, under uniaxial strain (open circles) and hydrostatic pressure (closed circles)

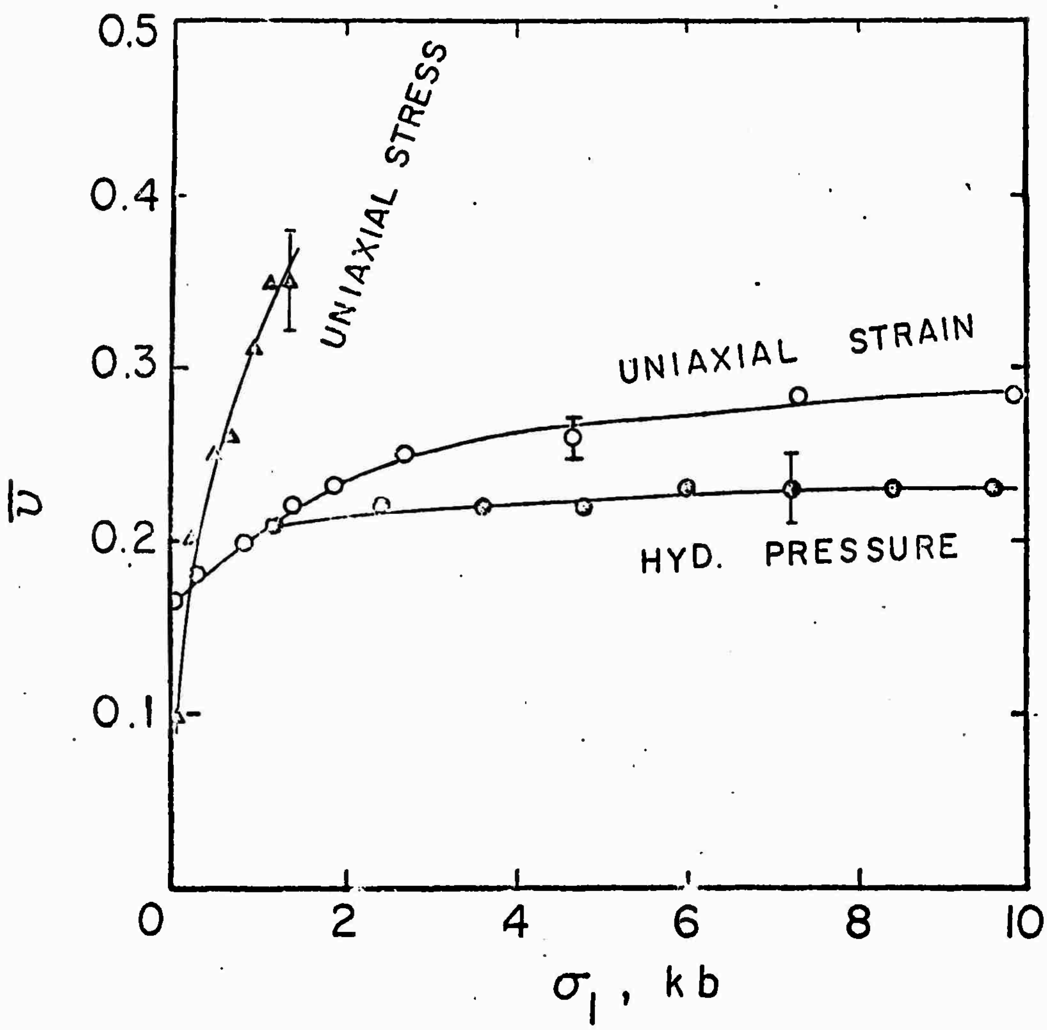

Fig. 4 Effective Poisson's ratio $\bar{\upsilon}$ for Westerly granite versus axial stress σ_1 under uniaxial stress [4], uniaxial strain [1], and hydrostatic pressure [1]

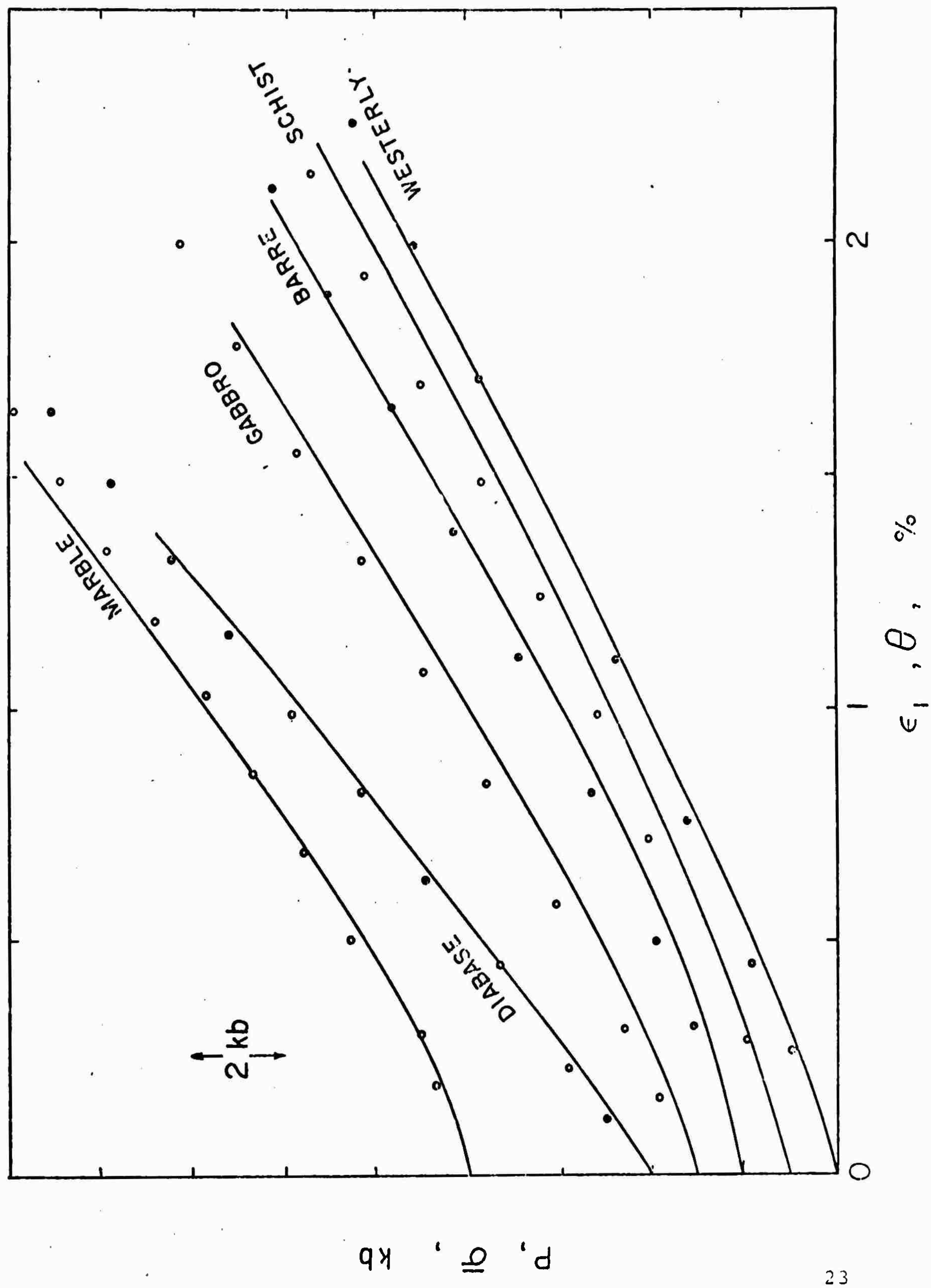

Fig. 5 Volumetric compression compared for hydrostatic pressure and uniaxial strain